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The saturation of galactic dynamos

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We summarize the current state of the long term discussion about the saturation mechanisms associated with rapid growth of small-scale magnetic field, that operate in large-scale galactic dynamos, and related problems with magnetic helicity conservation. Our general conclusion is that, taking into account magnetic helicity fluxes, large-scale magnetic field can be amplified up to about the equipartition level. In contrast, models without helicity fluxes give an initial temporal magnetic field growth, but then decay. In our opinion, it is more appropriate to refer to the situation as a “potentially catastrophic scenario” rather than as “catastrophic α -quenching”.

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1 Introduction

The formation of large-scale galactic magnetic fields is thought to be the result of dynamo action, based on the joint effects of differential rotation and the α -effect, the latter arising from the presence of mirror-asymmetric interstellar turbulence (e.g. Shukurov 2007). A straightforward application of this concept allows reproduction of the basic observational knowledge concerning magnetic fields in nearby spiral galaxies (e.g. Beck et al. 1996) and gives a more or less coherent scenario for the magnetic field evolution expected in the first galaxies to form (Arshakian et al. 2009). In particular, a very crude estimate of the equilibrium magnetic field, which is expected after the dynamo grows and is somehow saturated in the nonlinear regime, can be estimated based on the idea that the weak term of dynamo self-excitation, i.e. the α -effect, scales as

$$\alpha = \frac{\alpha_0}{1 + B^2/B_0^2}, \quad (1)$$

where α_0 is the hypothetical initial value of α which existed in the kinematic stage of dynamo action and B_0 is the magnetic field strength given by equipartition between the kinetic energy of turbulence and the magnetic energy. This estimate, although based on an obvious oversimplification, nevertheless gives a realistic hint to the possible order of magnitude and spatial structure of the dynamo generated magnetic field.

On the other hand the foundations of galactic dynamo theory are still however a topic of intensive debate. In particular, Cattaneo & Vainshtein (1991) suggested that whilst the standard formulation of the equations for the galactic dynamo is formally correct as a mathematical exercise, it

is physically irrelevant because it ignores dynamo saturation by small-scale magnetic fields, which grow much faster than the large-scale magnetic field (for a review see Kulsrud 1999). According to this idea, the presence of the small-scale magnetic field reduces the kinematic value of α -effect as

$$\alpha = \frac{\alpha_0}{1 + \text{Rm}^q B^2/B_0^2}, \quad (2)$$

where q is a constant of order unity, and Rm is the magnetic Reynolds number. Because $\text{Rm} = 10^6$ even if the estimate is conservatively based on ambipolar diffusion rather than Coulomb collisions (the complicated multiphase nature of the interstellar medium makes the appropriate estimate of Rm debatable) the scaling (2) is expected to lead to a saturation of the large-scale magnetic field at the level $B \sim B_0^{-q/2}$.

In contrast, the concept of the galactic dynamo which was considered that time as traditional, while appreciating that the growing small-scale magnetic field becomes dynamically important much earlier than the large-scale field, nevertheless believed that this would lead to a relatively minor effect, i.e. the dynamo saturates in a limited part of the turbulent spectrum which is unimportant for the mean-field dynamo as a whole (Ruzmaikin & Shukurov 1982; Ruzmaikin et al. 1988).

The idea of Cattaneo & Vainshtein (1991) has subsequently been addressed from various viewpoints, either with an explicit link to that paper, or quite independently (e.g. Kulsrud & Anderson 1993; Gruzinov & Diamond 1994; Cattaneo & Hughes 1996; Brandenburg 2001). A contemporary view of the problem can be summarized as follows. The galactic dynamo as a general concept survives and the traditional scheme with simple algebraic α -quenching as Eq. (1), although not based on very deep theoretical concepts, does lead to practical and useful models of dynamo

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action. The concerns of Cattaneo & Vainshtein (1991) are thus somewhat exaggerated. They did however lead to consideration of some finer features of galactic dynamos, which may be not very important for the pragmatic fitting of models to observations, but are nevertheless crucial for the physical understanding of the dynamo mechanism. The key development which came after 1991 is the appreciation that galaxies are open systems, and that fluxes of various quantities within and out of the dynamo active region are a crucial ingredient of the galactic dynamo mechanism. In principle, the role of such fluxes was stressed earlier by Zeldovich (e.g. Zeldovich et al. 1983), but its importance became obvious only later.

The discussion has been quite lengthy and spread over many papers as participants gradually refined their viewpoints, so to extract a coherent picture of the current situation from the literature requires some effort. The aim of this paper is to summarize the contemporary state of opinion about the problem, and to clarify to what extent the term “catastrophic α -quenching” gives an adequate description of the phenomenon discussed.

2 Magnetic helicity conservation

A weak point in the argument leading to both the opposing estimates (1) and (2) is that the growing small-scale magnetic field is considered as statistically mirror-symmetric, and that the magnetic energy density is a true scalar. Thus it is not very obvious how it can suppress the *pseudoscalar* quantity α . The argument becomes more convincing if based on another conserved quantity, namely magnetic helicity defined as

$$I_M = \int \mathbf{H} \cdot \mathbf{A} d^3x, \quad (3)$$

where $\mathbf{H}$ is magnetic field and $\mathbf{A}$ is its vector potential (see Kleeorin et al. 2000). This pseudoscalar quantity is an inviscid integral of motion of MHD and represents the total linkage of magnetic lines (e.g. Moffatt 1978).

Assume that the magnetic field was initially weak so that its magnetic helicity was also small, and separate the total magnetic field $\mathbf{H}$ and vector potential $\mathbf{A}$ into large- and small-scale components

$$\mathbf{H} = \mathbf{B} + \mathbf{b}, \quad \mathbf{A} = \bar{\mathbf{A}} + \mathbf{a}. \quad (4)$$

In the absence of helicity fluxes we obtain the balance relation

$$\mathbf{H} \cdot \mathbf{A} = \mathbf{B} \cdot \bar{\mathbf{A}} + \mathbf{b} \cdot \mathbf{a} = 0. \quad (5)$$

We will apply the above relation to a turbulent flow where it is natural to average the latter term over turbulent pulsations,

$$\mathbf{H} \cdot \mathbf{A} = \mathbf{B} \cdot \bar{\mathbf{A}} + \langle \mathbf{b} \cdot \mathbf{a} \rangle = 0, \quad (6)$$

where $\langle \rangle$ represents this averaging.

Taking into account that $\bar{A} \sim LH$ and $a \sim lb$ where L and l are the spatial scales of the large-scale and small-scale

magnetic fields respectively, that the galactic dynamo produces large-scale magnetic helicity $\mathbf{B} \cdot \bar{\mathbf{A}}$ which is of order of LB^2 and small-scale helicity $\langle \mathbf{a} \cdot \mathbf{b} \rangle$ of order lb^2 , and that the density of the small-scale magnetic helicity contributes to the suppression of the α -effect, we arrive at a scaling

$$\alpha \sim \frac{\alpha_0}{1 + LB^2/(lB_0^2)}. \quad (7)$$

An argument from balance relations based on the conservation law for I_M is less straightforward than with one using energy because of the gauge problem with $\mathbf{A}$. Nevertheless the plausible assumption that the large-scale magnetic field is axisymmetric and the small-scale field is locally homogeneous and isotropic (although mirror-asymmetric) resolves the gauge problem.

Catastrophic α -quenching in the form of Eq. (2) follows from Eq. (7), provided a scaling $l \sim LRm^{-q}$ is adopted. Such a scaling with $q = 1/2$ occurs for a small-scale dynamo in a short-correlated mirror-symmetric random flow (Kazantsev 1967; Zeldovich et al. 1990). This particular model is quite distant from the problem now being considered (there is no large-scale magnetic field, turbulent cascade, the magnetic helicity of the small-scale field vanishes, etc.). More relevant estimates of this type yield q somewhere between $1/3$ and $1/2$, depending on the particular model of turbulence (e.g. Rogachevskii & Kleeorin 1997).

It is important to stress that this conclusion from Eq. (7) so far ignores the real properties of interstellar small-scale magnetic fields as known from radioastronomical observations. Of course, the observations still contain some points that require clarification, but the general message looks quite clear (e.g. Beck et al. 1996): magnetic energy is more or less in equipartition with the kinetic energy of the interstellar turbulence, L is about several kiloparsecs while $l \approx 0.1$ kpc, and b/B does not exceed 2 (Beck et al. 2003). Thus the estimate $l \sim Rm^{-q}L$ with q large is clearly far from valid! Taking $l = 0.1$ kpc and $L = 3$ kpc we obtain from Eq. (7) the estimate

$$B/b \approx 1/5.5. \quad (8)$$

When compared with the observational result this is perhaps a little small for comfort, but not too severely so – certainly there is no catastrophic quenching. The more serious problem is that this saturation is approached on a diffusive timescale, i.e. depending on the microscopic diffusion coefficient, which is much longer than a galactic age – see for example Sect. 8.5.2 of Brandenburg & Subramanian (2005).

Our main conclusion is that the fact that we observe $l/L \sim 1/30$ tells us that the catastrophic quenching scenario is not operating, and so we deduce the presence of helicity fluxes. Thus any model that does not include fluxes will not reproduce the observations. This is confirmed by the observational estimate that $B/b \sim 1$.

3 Dynamo models with helicity fluxes

Shukurov et al. (2006) investigated the effects of a simulated galactic fountain flow, which preferentially removes

small scale helicity from the disc region. They showed that in a simple local galactic dynamo model, the field can grow to saturation levels in a turbulent diffusion time, i.e. in a time comfortably shorter than the galactic age. At saturation, the large-scale field strength was as large as 10% of the equipartition value. This model is grossly simplified, but clearly and simply shows the potential of helicity fluxes to resolve the saturation time problem.

There are a number of formulations of an evolution equation for the α -coefficient, more accurately for the magnetic helicity. This approach is known generically as “dynamical α ” and includes work by Vishniac & Cho (2001), Kleeorin et al. (2000, 2002, 2003, and references therein), and Sur, Shukurov & Subramanian (2007). Although these studies differ in detail, the general thrust is similar, and here we will examine the formulation and results of the work by Kleeorin et al. (2002, 2003), in which they specifically address the galactic dynamo problem.

In Kleeorin et al. (2002, 2003), the total nonlinear α -effect is written as

$$\alpha(\mathbf{B}) = \alpha^v + \alpha^m, \quad (9)$$

where α^v , α^m are, respectively, the hydrodynamic and magnetic parts of the α -effect. The later is governed by a differential (rather than algebraic) equation which includes the flux $\mathbf{F}$ of magnetic helicity. The equation is quite lengthy and we give here the steady state form

$$\alpha = \alpha^v - \frac{4 \text{Rm} \phi}{3} [\alpha B^2 - \eta_T (\mathbf{B} \cdot \text{curl} \mathbf{B} - 9\pi\rho \nabla \cdot \mathbf{F})], \quad (10)$$

where η_T is the turbulent diffusivity, ρ the density, and $\phi(|\mathbf{B}|)$ is an algebraic quenching function. This yields an estimate

$$\alpha = \frac{\frac{4}{3} \text{Rm} \phi_m \eta_T (\mathbf{B} \cdot \text{curl} \mathbf{B} - 9\pi\rho \nabla \cdot \mathbf{F})}{1 + \frac{4}{3} \text{Rm} \phi_m B^2}. \quad (11)$$

For large Rm ,

$$\alpha \rightarrow \eta_T \frac{\mathbf{B} \cdot \text{curl} \mathbf{B} - 9\pi\rho \nabla \cdot \mathbf{F}}{B^2}, \quad (12)$$

which is independent of Rm .

Dynamical quenching based on magnetic helicity conservation (in addition to simple algebraic quenching) can be embedded in a local model for the galactic dynamo (e.g. Ruzmaikin et al. 1988) as well as in the “no- z ” model (Subramanian & Mestel 1993; Moss 1995) which considers fields to be averaged vertically across the disc, i.e. in the z -direction (Kleeorin et al. 2002, 2003).

Results of numerical simulations with the model can be summarized as follows (we assume that a realistic set of the dynamo governing parameters is chosen):

1. Magnetic fields in a model with dynamical α -quenching but without helicity fluxes ($\mathbf{F} = 0$) grow initially and then decay rapidly to very small values.
2. The magnetic field in a model with dynamical α -quenching and helicity fluxes grows, and then smoothly saturates near to equipartition between magnetic and turbulent energies. The evolution is quite similar to that

Fig. 1 Results showing the evolution of the midplane field without helicity fluxes (long-dashed); with the fluxes driven by anisotropies in turbulence (solid) as Kleeorin et al. (2002, 2003); and with the addition of an explicit advection of flux in the z -direction (short-dashed).

found with simple algebraic α -quenching – Eq. (1). If the dynamo action is too strong, dynamical α -quenching based on simple helicity fluxes may be insufficient and more effective saturation mechanisms such as buoyancy may then play a role.

Given these conclusions, we feel that the terminology “potentially catastrophic scenario” is perhaps more appropriate for this situation than “catastrophic α -quenching”.

A formulation of the dynamical α -effect can, of course, be combined with a naive advection of helicity by, e.g., a fountain flow. Simple models (e.g. Sur et al. 2007) show that the equilibrium field strength can be enhanced in such a situation – see, for example, Fig. 1, which is derived from the model of Kleeorin et al. (2002, 2003).

4 Discussion and conclusions

We conclude that the mean field galactic dynamo is viability and able to incorporate relevant ideas from the criticism of Cattaneo & Vainshtein (1991). In practice, traditional galactic dynamo models based on a simple algebraic α -quenching appear adequate for a pragmatic fitting of models to astronomical observations. The differences between the predictions of such traditional models and those from models with dynamical quenching and helicity fluxes appear insignificant in comparison with uncertainties in the dynamo governing parameters and the physics omitted from the modelling.

Dynamical α -quenching is a promising alternative to DNS for galactic studies (or even DNS can be considered as a helpful complement to the α -quenching concept). In particular, recent DNS of SN driven turbulence in the ISM support the idea of an advective transport of magnetic helicity while the opposite action of the turbulent pumping of

the mean field avoids the dynamo suppression by the wind (Gressel et al. 2008). There are several different treatments and mechanisms, which are generically similar. The idea of a naive flux driven by fountain flows (as discussed) and/or galactic winds should not be dismissed.

The balance equations for magnetic helicity seem now to be understood in a broad sense. An investigation of the magnetic helicity distribution over the turbulent spectrum in context of the helicity balance between small- and large-scale fields looks as a natural further step towards understanding galactic dynamos. First steps in this direction have been made by Frick et al. (2006) and Stepanov et al. (2008).

The discussion here has been explicitly in the context of galactic dynamos. This is partly because, in comparison with solar and stellar dynamos, these systems appear relatively simple. Moreover galaxies are more-or-less transparent, and so we can observe magnetic and velocity fields through much of their volumes, in some contrast to the stellar case. However there are generic similarities between stellar and galactic systems. In particular, like galaxies, the Sun and stars are open systems with coronal ejections, solar and stellar winds, etc. The associated helicity fluxes may be correspondingly important, and their effects deserve to be studied (see e.g. Zhang et al. 2006).

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