situation. Thus many sections of the book focus on integrable equations.

After a brief introduction, the authors start by introducing the Painlevé test. Their goal is not to obtain a "yes" or "no" answer, but rather to get as much detailed information as possible about the Laurent expansions of all meromorphic solutions of the equation at hand. It is this information that is used as input for the integration method of the third chapter: using a beautiful result of Briot, Bouquet, Fuchs, Poincaré, and Painlevé, an algorithm is described to find, in closed form, all solutions of a differential equation which are elliptic or limits of an elliptic function. The algorithm encompasses other algorithms which assume an ansatz for the solution form that is far more restrictive. I am so enthused with this algorithm that I have used it already to obtain closed-form solutions for two equations where I had previously failed to do so. One of these is not integrable! Furthermore, I have tracked down a copy of Valiron's text [4], which contains a great exposition of the classical results (no longer taught, of course) that enable this algorithm and more. Many of the ideas of Chapters 2 and 3 are extended in Chapters 4 and 5, where they are used directly on partial differential equations. At this point partially integrable equations disappear into the background, as integrable equations are emphasized and many techniques and concepts that are introduced do not apply to the partially integrable case. The authors introduce the concepts of a Lax pair, Darboux transformations, Bäcklund transformations, etc., in Chapter 5. The exposition is self-contained, but I would not recommend it as a first introduction to these ideas: their setting is too restrictive here and the bigger picture of their importance would be missed. Chapters 6 and 7 deal with the explicit integration of Hamiltonian systems of Hénon–Heiles form and discrete equations, respectively. Both chapters are informative but of interest to a smaller audience than the rest of the book, in my opinion. Some appendices with useful background and reference material round out the book.

Overall, the book is very well written. It is clear that the authors mean for the book to be accessible, and they have succeeded. The book is sprinkled with the types of sometimes offbeat comments that one would encounter in a research meeting, but usually never read in a book. For instance, very often the authors comment on the importance of specific results or of general approaches that are found in the literature. Such comments are never thrown in gratuitously: reasons and context are always added. A subjective writing style like this may offend some readers, but I rather enjoy reading authors who have a clear opinion on their subject matter. In summary, Conte and Musette have written an excellent introduction to some of the methods Painlevé and his collaborators used and, more importantly, to how those methods are still relevant today. I highly recommend their handbook.

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Bernard Deconinck
University of Washington

Functions of Matrices: Theory and Computation. *By Nicholas J. Higham.* SIAM, Philadelphia, 2008. $59.00. xx+425 pp., hardcover. ISBN 978-0-898716-46-7.

It has been a pleasure to read this book and be asked to review it. The book *Functions of Matrices: Theory and Computation*, by Nicholas J. Higham, is not just a good book, it is a beautiful new page in the long history of numerical linear algebra, history to which the author continues to contribute as we summarize in this review his latest accomplishment. Nick Higham has the wonderful gift of being able to write in a remarkably clear and precise way about the results of his deep understanding of numerical linear algebra. In the book under review, Higham's love for the subject comes across loud and clear, and it is contagious. The author does not just describe the methods, his effort is (as it has always been) *to show how to interpret the behavior of the methods.* No one else is doing this at the level of Higham, and we are grateful.

The scope of the book *Functions of Matrices* is to provide a comprehensive treatment of the subject, and the author covers both introductory material and advanced topics. In our opinion the book gains its scope in a very pleasant way, always maintaining a perfect equilibrium in all its parts. The exposition is sufficiently detailed to be complete and self-contained, without sacrificing synthesis and readability. In this book, we still find the author's insight into finite precision behavior of algorithms, just like in his previous book, *Accuracy and Stability of Numerical Algorithms*, also published by SIAM. But here, the author also utilizes tools which are more dynamical in nature, and that better reflect the fact that the problem under scrutiny is inherently represented by an infinite-dimensional process, so that the behavior of iterations becomes crucial.

In the early 1990s, when we started to work on matrix functions in general and on the logarithm in particular, we were aware that a lot of numerical work on this subject still had to be done. Functions of matrices had been understood, in their mathematical meaning, for a long time, and the theory of functions of matrices had been adequately covered in several books: among them, *Topics in Matrix Analysis* by Horn and Johnson remains a model of clarity. However, from the computational point of view, much more work was needed. Some instances of specific functions were already better understood, namely, the exponential, which is surely the most famous and studied matrix function; the famous paper "Nineteen Dubious Ways to Compute the Exponential of a Matrix," by Moler and Van Loan, is dated 1978. Some years later, the matrix sign function and the matrix square root function attracted the interest of several researchers, among them the author of this book himself. But a complete understanding of other functions, like the logarithm or the sine and cosine, as well as the indication of a reliable procedure to compute a general matrix function, were still lacking. Indeed, in the 1990s, the MATLAB implementation of the command `logm`, for the matrix logarithm, and of the command `funm`, for the computation of a general matrix function `fun`, often gave very inaccurate results, being based on unstable procedures. Ever since then, a lot of work on numerical computation of functions of matrices has been done, both to develop reliable numerical techniques and to provide high quality software for functions of matrices. The author of the book under review has devoted a lot of effort not only to the theory, but also to the implementation of reliable numerical procedures. The MATLAB "Matrix Function Toolbox" was written by N. Higham, and the earlier pitfalls of this part of MATLAB are now fixed. Interspersed within the book are the results of the choices that the author implemented in the MATLAB toolbox, choices backed up by theoretical analysis and by extensive testing of different strategies, to produce an end result which is more stable and efficient than previous choices. As a result, the most important functions, `expm`, `logm`, and `funm`, are all new in MATLAB.

This book is the natural outcome of all these efforts. It is particularly welcome because it is the first one entirely devoted to the subject and because interest in the computation of matrix functions has dramatically increased in the last twenty years.

The first part of the book (Chapters 1–4) is an introduction to the problem of defining and computing a general matrix function. Chapter 1 contains the main theoretical results that are needed in the rest of the book. It also contains a brief history of matrix functions, particularly how to define a matrix function. Indeed, three different definitions can be found in the literature. The oldest one is based on Hermite polynomial interpolation and is due to Sylvester (1883). Then Frobenius (1896) and Poincaré (1899) proposed the representation of a matrix function through the Cauchy integral theorem. The third definition, via the Jordan canonical form, is due to Giorgi (1928). Relative limits and merits of these three equivalent definitions are considered. In Chapter 2, the author describes several problems, coming from applications, where one needs the computation of logarithms, square roots, and other functions of matrices, besides the well-known exponential. In Chapter 3, the sensitivity of matrix functions is addressed, with a central role played by the Fréchet derivative. The main strategies to estimate the condition number of a matrix function are also discussed. Chapter 4 is dedicated to an overview of the most important techniques applicable to a general function: polynomial and rational approximations; Schur reduction to triangular form followed by a (block) Parlett recurrence; and matrix iterations, enriched by a stability analysis.

The second part of the book (Chapters 5–14) traces the state of the art of the research in the field, with great attention paid to classical issues like accuracy, stability, and computational cost. One full chapter is devoted to each of the functions commonly used in applications: sign, roots, exponential, logarithm, and trigonometric functions. Particular relevance is given to the matrix sign function, both in Chapter 5, which is explicitly dedicated to it, and through many connections with the matrix square root in Chapter 6, the matrix pth root in Chapter 7, and the polar decomposition in Chapter 8. What joins these apparently unrelated problems is the possibility of solving them via matrix iterations that are somewhat equivalent.

The best general approach to compute a matrix function is based on a Schur decomposition with ordering of the eigenvalues, followed by the computation of the function of the diagonal blocks of the matrix and by a block Parlett recurrence to recover the off-diagonal part. In our computation on logarithms of matrices, we had also implemented this strategy and can easily confirm its superiority with respect to other techniques, as a general purpose approach. In the present book, this approach is discussed in general in Chapter 9 and analyzed in particular for the exponential, logarithm, and sine and cosine functions (in Chapters 10, 11, and 12, respectively). For these transcendental functions, Padé approximations and suitable strategies of scaling and squaring are also reviewed.

Chapters 5 to 12 of the book, albeit general in theoretical scope, are of computational interest for the case when the entire matrix $f(A)$ is computed, and thus when A is not large. For large matrices A, presumably sparse, one is typically interested in the action of $f(A)$ on a vector b. Evaluation of $f(A)b$ is addressed in Chapter 13. This chapter is shorter than the others, since computation of $f(A)b$, for A large, is still the object of current research interest. Finally, miscellaneous issues, like the computation of structured matrix functions for structured matrices, are mentioned in Chapter 14.

The book is very well written and carefully organized. The literature review is informative. All arguments are treated with the clarity and rigor that are typical of the author. Though nontrivial in itself, the material contained in the book does not require knowledge of sophisticated mathematical tools; it can be understood and appreciated by a reader with "a basic grounding in numerical analysis and linear algebra" (quoting from the preface). Moreover, the book is enriched by a large number of examples, exercises, and open questions. At the end of each chapter there are notes, hints, and references. For the above reasons, the book is not only a mandatory reference for researchers in the field, but it is also well suited for a graduate course on functions of matrices.

To conclude, we can only reiterate our appreciation for this new page of numerical linear algebra history, and we recommend the book to anyone with either a theoretical or a computational interest in functions of matrices.

LUCA DIECI
Georgia Institute of Technology

ALESSANDRA PAPINI
University of Florence, Italy

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