development is done for general bornological algebras. In particular, the authors present a proof, due to Ralf Meyer, of a version of Nigel Higson's theorem on automatic homotopy invariance for stable split-exact functors, in the category of bornological algebras. Other important properties of K-theory for C*-algebras, such as Bott periodicity and the Pimsner–Voiculescu exact sequence, are established for local algebras, while Connes' Thom isomorphism for crossed products by $\mathbb{R}$ is presented in its original generality, for C*-algebras only.

In its treatment of bi-variant K-theories, which may be considered as generalisations of (parts of) the KK-theory of Kasparov for C*-algebras, the book follows some of the approaches developed by Joachim Cuntz over the last 25 years, but here in the setting of bornological algebras. In this generality one of the book's chapters describes the construction of the universal triangulated homotopy theory, which is used in the following chapters to define various new bi-variant K-theories for bornological algebras and compare these theories with other existing theories for Banach algebras and Fréchet algebras. It is clear that an important driving force for the authors has been the desire to extend the powerful machinery of K-theory and KK-theory from the category of C*-algebras to more general categories of algebras, and it is interesting to see where, to what extent, and how this is sometimes possible; where the efforts meet serious resistance; and where the desired generalisations are downright impossible. The book serves as a good introduction to the authors' work in this direction and can partly be seen as an account of the present status.

The book touches on a series of other aspects and applications, such as twisted K-theory, equivariant theories, continuous-trace C*-algebras, Takai duality, index theory, pseudo-differential operators and the universal coefficient theorem. And this is far from a complete list of subjects which are discussed to varying degrees in the book. It is thought-provoking that many of the potential readers of this review could each add to the list of important aspects of topological and bivariant K-theory which are not mentioned in the book. An example would be the application of K-theory and KK-theory to the classification of C*-algebras, but this is purportedly a different story.

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Functions of matrices: Theory and computation

By Nicholas J. Higham: 425 pp., US$59.00, ISBN 978-0-898797-80-7
(Society for Industrial and Applied Mathematics, Philadelphia, 2008).

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In his review of Nick Higham's previous text *Accuracy and stability of numerical algorithms* [**1**], Pete Stewart [**2**] says:

> Nick Higham has assembled an enormous amount of important and useful material in a coherent, readable form. His book belongs on the shelf of anyone who has more than a casual interest in rounding error and matrix computations. I hope the author will give us the 600-odd page sequel. But if not, he has more than earned his respite — and our gratitude.

Nick Higham has fallen short of 600 pages, but I am sure this book will join his other books on the shelves of many specialists in linear algebra, numerical analysts and practitioners.

The book provides a review of historical results, details of research of recent vintage (much of it by the author) and a number of new theorems. In all places, simple examples illustrate key points, and these make understanding the information a great deal more straightforward. Each chapter closes with historical notes and references, and a set of problems. This adds a very human touch to the book, and makes it a more satisfying read as well as providing respite from the mental activity needed to read mathematics. The problems (with solutions in an Appendix) form an integral part of the book, and contribute to making this an ideal graduate text. New research problems are also formulated, which will allow anyone inspired by the text to continue working in this area.

The first four chapters form a basic introduction, with general techniques. Chapters 6 to 12 deal with the specific matrix functions *sign*, *square* and *pth roots*, *exponential*, *logarithm*, the *Schur–Parlett algorithm* and *polar decomposition*. The final two main chapters discuss the computation of $f(A)b$, where b is a vector, and miscellaneous results which are of general interest but fit nowhere else. There are further appendices containing notation used in the book, basic definitions and results from linear algebra and numerical analysis, operation counts for standard matrix computations, and MATLAB implementations of algorithms described in the book.

The first chapter discusses the notion of a function f of a matrix A, and gives definitions via Jordan canonical form, Hermite interpolation, and the Cauchy integral formula. The key idea behind the first two of these is that $f(A)$ is determined by its values (and the values of its derivatives) on the spectrum of A, i.e., by a discrete set of values. So $f(A) = p(A)$ for some polynomial p. One must keep in mind that the polynomial depends both on f and A.

The second chapter provides a number of applications, demonstrating the ubiquity of the problem of computing functions of a matrix. In particular the matrix *sign* function is introduced. The importance of this function for computing, for instance, the matrix square root and the polar decomposition, is emphasised in the introduction to the book, and Chapter 5 is given over to the computation of this function.

Chapter 3 looks at the issue of *conditioning*, which refers to the sensitivity of algorithms to small changes in the input data. The derivative of the function is clearly important here and as we are looking at a high-dimensional input space the Fréchet derivative is the appropriate tool. Properties of this derivative are discussed, and estimates for the condition number of the computation of f are given.

In Chapter 4 a number of general techniques are introduced and these are used in Chapters 5–12 in dealing with specific functions or algorithms. These techniques include computation of matrix powers and polynomial evaluation, rational (including Padé) approximation, the Schur decomposition and matrix iteration.

I am not an expert in numerical linear algebra, so it is not for me to say what place this book will occupy in that area. However, I would heartily recommend that this book be read by any seasoned numerical analyst and graduate student, as it is full of interesting and applicable information, and is as easy a read as a book of this level can be. I can only echo the thanks of Pete Stewart for Nick Higham's masterful summary of the theory and computation of functions of a matrix, and feel sure that this book will serve to inspire many others to work in this area of fundamental importance to all numerical scientists.

References

1. N. J. HIGHAM, *Accuracy and stability of numerical algorithms* (Society for Industrial and Applied Mathematics, Philadelphia, PA, 1996).
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